1. Goals

A. Physics

In this experiment you will measure the strength of a magnetic field using two methods. The first is dubbed the "flipped coil" technique, and is based on Faraday's law of induction- By moving a coil through a magnetic field you will generate measurable currents from which you will obtain the field strength. The second method is the current balance method, in which you will measure the Lorentz force on a current carrying conductor in a magnetic field. From the measured magnitude of the force you will be able to deduce the magnetic field.

B. Technique

- Gain some experience with the basic *RC* integration circuit.
- Use the oscilloscope as a charge indicator at slow sweep speeds.

C. **Questions**

To be worked out in your lab book before coming to the lab session

- If the flip-coil has 100 turns and a 4 cm^2 area, how much charge can be stored on the capacitor of Fig. 2 for a magnetic field of 0.5 Tesla and resistance of $10^6\ \Omega$? What voltage is this on a 2 μF capacitor?

D. References

- Halliday and Resnick, 3rd Ed, Ch. 30-7; 31-2, 3; 32-3; 34-8
- Young, University Physics, 8th Ed., Ch. 28, 29, 30

2. Background and Theory

A. Introduction

In this experiment we examine the measurement of an unknown magnetic field by two different methods. These methods involve the use of two laws of electromagnetism that you have already seen in your earlier work, namely Faraday's law of induction, and the force law for the interaction between a current and a magnetic field.

The appropriate laws will be restated for your use, and it will be assumed that you understood their derivation in your first year lecture course, or, in the unhappy event that this is not so, that you will consult a textbook for the missing details. There is considerable extra detail associated with the induction measurement; this also is an introduction to the use of an *RC* circuit as an integrator.

In MKS units, Faraday's law is:

$$V = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = -\frac{d\Phi}{dt} \tag{1}$$

where, you may recall, V is the integral $\int \mathbf{E}\cdot d\ell$ around a closed contour, or in a practical sense, the voltage around a circuit, while Φ is the total magnetic flux passing through the surface contained by the circuit. The minus sign came from Lenz's Law (and we'll drop it for the rest of this section), since the induced currents oppose the magnetic flux change.

The force on a current element $I\,d\ell$ due to a magnetic field B is:

$$d\mathbf{F} = I\ d\ell \times \mathbf{B} \qquad \textbf{(2)}$$

where the appropriate units are: V (Volts), Φ (Tesla-m^2=Weber), F (Newtons), I (Amperes), B (Tesla), and length (m).

B. Flip-Coil Technique

Having written down the essential mathematical relationships between magnetic field, potential, and current, we must now ask how we can use these to measure a constant magnetic field. Consider Faraday's law first. It relates a circuit voltage to the time derivative of magnetic flux though it; yet our problem is the measurement of a steady field which has no time variation at all. Furthermore, even if the field were varying, we are interested in its magnitude, not its rate of increase.

Let us arrive at a technique by considering the above two objections in reverse order. How do we find the magnitude of B (or Φ) from its derivative? By integrating it, obviously. But how does one integrate a voltage from an electrical circuit? There are several ways, actually, but the very simplest is one of the best. Imagine the electrical network shown in Fig. 1.

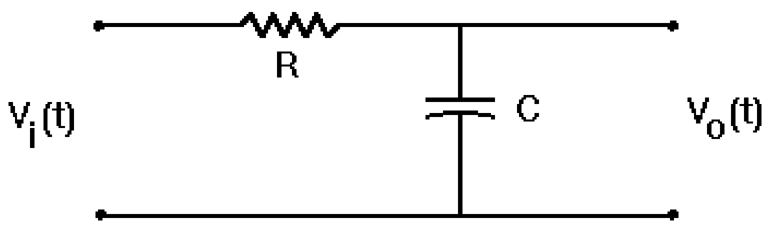

Figure 1 *An RC integrator.*

We will impress a time varying voltage $V_i(t)$ at the input terminals at the left, and look at the resulting output voltage $V_o(t)$ at the right.

Recall first that the voltage across a capacitor is proportional to the total charge Q placed on it, where the constant of proportionality is the capacitance, C. Thus,

$$V = \frac{Q}{C} \qquad \textbf{(3)}$$

Since current, I, is a rate of charge transfer, it follows that:

$$Q(t_1) = \int_0^{t_1} I dt \qquad \textbf{(4)}$$

if Q = 0 at t = 0. Then,

$$V(t_1) = \frac{1}{C}\int_0^{t_1} I dt = V_o(t_1) \qquad \textbf{(5)}$$

since our output voltage is just the voltage across the capacitor. Now what is I? Ohm's law tells us that:

$$I = \frac{V_i - V_o}{R} \qquad \textbf{(6)}$$

and so by direct substitution into Eq. 5,

$$V_o(t_1) = \frac{1}{RC}\int_0^{t_1}\left[V_i(t) - V_o(t)\right]dt$$
$$= \frac{1}{RC}\int_0^{t_1} V_i(t)dt - \frac{1}{RC}\int_0^{t_1} V_o(t)dt \tag{7}$$

If the product RC, which itself has the dimensions of a time, is much larger than the integration time t_1 (that means that V_0 varies slowly on time scales of t_1) the second right-hand term becomes negligible- we approximate $\int_0^{t_1} V_o dt$ by $V_o t_1$, then,

$$V_o(t_1) = \frac{1}{RC\left(1+\frac{t_1}{RC}\right)}\int_0^{t_1} V_i(t)dt \approx \frac{1}{RC}\int_0^{t_1} V_i(t)dt \tag{8}$$

where in the last step we have used the fact that $t_1 / RC << 1$. From Eq. 8 we see that our circuit acts an <u>integrator</u>. The output voltage at any time t_1 (provided that it is smaller than RC) is the time integral of the input voltage.

We can now connect such an integrator to a loop of wire located in a time-varying magnetic field increasing from $B = 0$, and obtain an output V_o which is related to the flux through the loop by (ignoring the negative sign, see Eq. 1)

$$V_o(t) = \frac{1}{RC}\int_0^{t}\frac{d\Phi}{dt}dt = \frac{\Phi(t)}{RC} \tag{9}$$

as long as $RC >> t$. If the loop has an area A, and the field strength B is <u>uniform</u> over this area, then $\Phi = AB$, and

$$V_o(t) = \frac{A}{RC}B(t) \tag{10}$$

We can measure a magnetic field that has increased from zero in some suitable short time t. But how about measuring a steady field? You may already have anticipated the answer to this. We will simply take our loop which initially rests in a place where there is no field and thrust it suddenly into the magnetic region. By "suddenly", we mean in a time much shorter than RC. The loop itself then sees a sudden rising field, and develops an appropriate voltage at its terminals. In fact, in most cases it is more practical to have the coil at rest within the field, and then suddenly pull it out. The total <u>change</u> in B, which is the initial B itself, will appear as a change in V_o (= ΔV_o) at the integrator output. Fig. 2 illustrates our scheme, which is called the <u>flip-coil technique</u>.

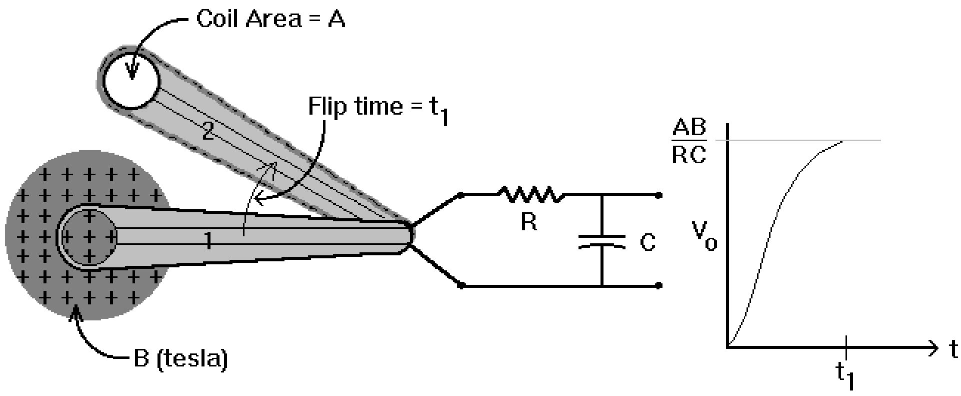

Figure 2 *Illustration of the setup for moving a flip-coil out of a magnetic field, while integrating the induced currents with an RC circuit*

Sometimes the V_o we can obtain (while still satisfying the requirement that $RC >> t_1$) is inconveniently low. This is easily remedied by wrapping several turns, say n of them, on the loop. The output increases by a factor of n, since this is just equivalent to connecting n separate single-turn flip coils in series and thus adding their output voltages. In this case, then,

$$\Delta V_o = \frac{nA}{RC} \Delta B \qquad \textbf{(11)}$$

What happens, now, when the "flip" has occurred, and the integrator output voltage has jumped to ΔV_o? We can answer this by taking another look at the circuit, as shown in Fig. 3.

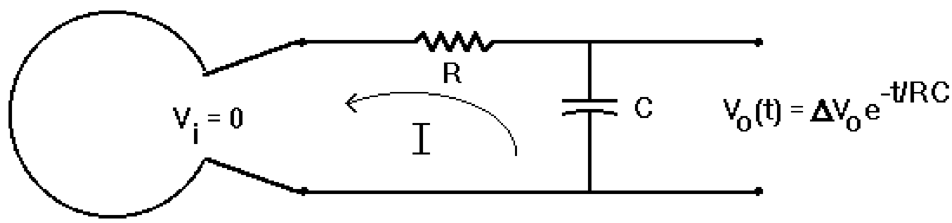

Figure 3 *The RC integrator, discharging through the loop.*

There is no more input voltage, since $dF/dt = 0$ in the coil; the capacitor simply discharges through R and the coil, whose resistance is usually negligible. You may recall that when a capacitor discharges through a resistance, its voltage decays as

$$V(t) = V_o e^{-t/RC} \qquad \textbf{(12)}$$

We conclude then that the whole history of the integrator output voltage must look something like that shown in Fig. 4.

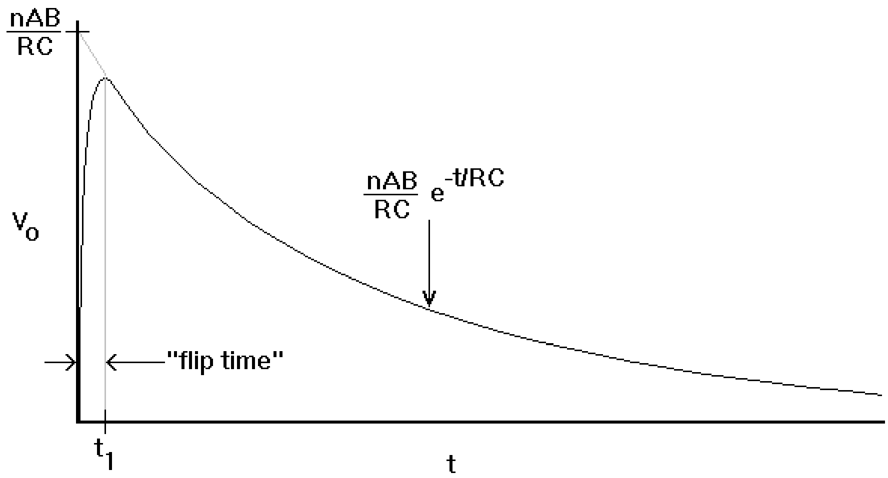

Figure 4 *Time history of the output voltage for a flip coil moved rapidly out of a magnetic field region and integrated by an RC circuit followed by discharge of the capacitor*

We can see now in what sense the flip of the coil must be sudden. If the discharge rate of the capacitor is anywhere near the charging rate during the flip of the coil, the output voltage will never reach it proper level. We must, then make t_1 much less than the discharge time constant which is RC; but this is simply restating our criterion for accurate integration by our integrator circuit.

There is one more experimental "fact of life" to take into account in this system. It is that the device which measures $V_o(t)$ has something less than infinite resistance itself. The input resistance of the oscilloscope you will use is 10^6 ohms, for example. The complete circuit for our measurement is then:

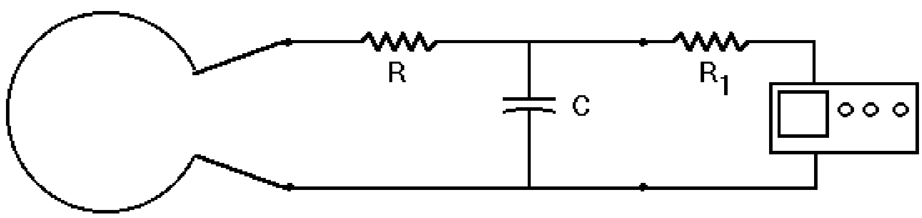

Figure 5 *The flux integrator, with the oscilloscope input resistance shown as R_1.*

It is clear that R and R_1 are in parallel as far as the discharge of the capacitor is concerned, and so the decay time constant is R_2C, where:

$$R_2 = \frac{R_1 R}{R_1 + R} \qquad \textbf{(13)}$$

But what about the integration time constant? The answer is that this is still RC because the capacitor is charged only through R, and so

$$\Delta V_o = \frac{nA}{RC}\Delta B \qquad \textbf{(14)}$$

where now the flip time t_1 must be kept small compared to R_2C. You should try to prove this to yourself.

C. Current-Balance Technique

The measurement of B by measuring the force on a current in the field is so straightforward as to hardly require an elaborate introduction at all. The only subtlety involved in such an experiment lies in the means by which one makes sure that the force that is measured is applied only to a well-defined segment of conductor in a homogeneous region of the field.

Suppose we were to try a measurement of the field between the poles of a magnet where the field has a typical distribution as shown in Fig. 6.

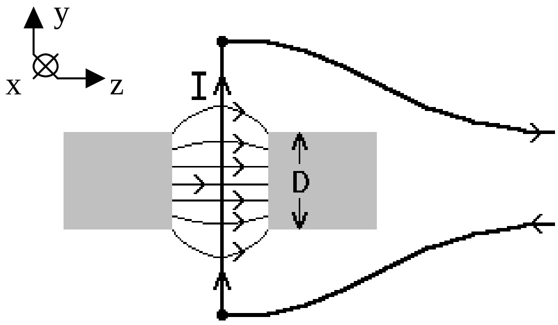

Figure 6 *Current loop passing through the magnetic field created by a dipole magnet*

We draw "field lines" to represent *B*, as has become conventional, where the density of lines is a measure of the local field strength. It is characteristic of this field distribution that the magnitude of *B*, i.e., the line density, becomes smaller in the "fringing" region, and falls gradually from its maximum value between the poles to zero far outside.

A wire carrying current *I* in the y direction, placed in a magnetic field in the z direction, will then feel a total force in the x direction:

$$F_x = \int I_y B_z(y) dy \qquad \textbf{(15)}$$

However, this doesn't tell the experimenter what *B* itself is at any point; we can only determine the integral $\int B_z \, dy$. If *B* were perfectly constant over the width of the pole faces and dropped abruptly to zero at the edges, then things would be simpler, and the integral would be $B_z D$, where *D* is the pole diameter; the field would then be given by

$$B_z = \frac{F_x}{I_y D} \qquad \textbf{(16)}$$

A strategy we might employ in this experiment would have *I* flow only in the homogeneous region of the field. A moment's thought exposes this as nonsense, however, since the current has to enter from the outside and return there again.

A more realistic and workable strategy would be to arrange the wire so that the forces experienced by the wire as it enters and leaves through the inhomogeneous part of the field (i.e., the fringing field) cancel each other out, leaving only a net force from a section of the wire in the uniform field. How can these entering and leaving forces be made to cancel? Remember that the vector relation

$$dF = I \ d\ell \times B$$

is resolved into:

$$dF_x = I_y B_z d\ell \qquad \textbf{(17)}$$

in the coordinates of our example. Now, B_z does not change sign; therefore, if we want to generate two forces having opposite signs in the fringing field, we can only do so by using two oppositely directed currents. You should be ready by now to appreciate the unique properties of the arrangement of wire in the region of the magnet poles shown in Fig. 7.

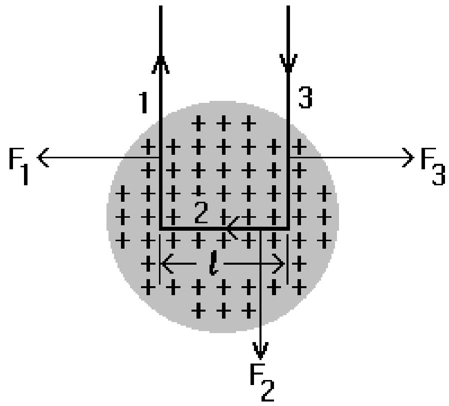

Figure 7 *Forces on a hairpin circuit with current I, placed in a magnetic field as seen by looking along a line of B towards a pole face*

The wire has the form of a hairpin, with three distinct straight segments, labeled 1, 2, 3, in the field. It should be clear that since *I* is equal but oppositely directed in legs 1 and 3, and since these pass through identical field distributions, their forces, F_1 and F_3 are also equal and opposite, and so add to zero.

All that remains is leg 2 of length *L,* which is short enough to be everywhere in a uniform field. Therefore, the total force on the hairpin is

$$F = IB\ell \qquad \textbf{(18)}$$

and one may now measure *B* by, for example, hanging the hairpin from a balance so as to measure F_2, and sending some carefully measured current *I* through it. This is precisely what you will do in this experiment.

3. The Experiment

A. Purpose

To measure the strength of a magnetic field by two methods based on different physical principles: Faraday's Law and forces on a current element.

B. Equipment

1) Permanent magnet assemblies;
2) Current hairpins of various lengths;
3) Resistor box;
4) Capacitor board;
5) Flip coil on wand;
6) Oscilloscope;
7) Electronic balances (.01 gm and .1 gm resolutions);
8) Small power supply with separate 0-5 amp meter for use with the current hairpin;
9) Centimeter ruler.

C. Apply the Force Measurement Technique

Measure the magnetic field of a permanent magnet assembly by measuring the force on the magnet as a function of the current in the hairpin between the pole faces, as shown in Fig. 8.

Measure the force on the hairpin for several different currents, and do so for several hairpins having different lengths ℓ. To do so, place magnet (partly painted red) on the balance and weigh it. Then zero the balance by pushing the "tare" button, so that the balance will only display changes from this starting weight. Don't forget to convert your values from weights to forces.

Before you start collecting data, make a rough check with moderate current to see that the direction of the current in the hairpin circuit produces a downward force, and therefore an increase in the weight measured by the scale. If not, reverse the leads at the main unit or invert the magnet direction.

Make a plot showing F as a function of $I\ell$, note that all your data should fall on the same line in this plot. Perform a linear least-squares fit to your graph, and determine a value of the magnetic field B (with uncertainty) from the slope as shown in Eq. 18.

You will find in the lab an additional magnet; it is black and has a circular cross section. This is the magnet that you will measure next using the flip coil technique. Use the current setup to perform a force measurement for this magnet, so that you will be able to compare your results to those of the flip-coil method. Note that you will have to use a different balance with this heavier magnet. In addition, use a current hairpin whose ℓ is sufficiently short to fit within the homogeneous region of this magnet.

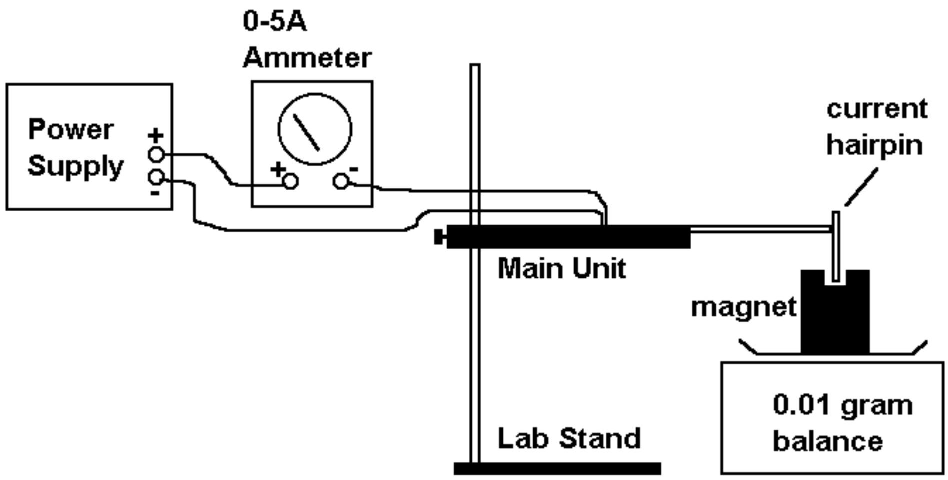

Figure 8 *Schematic view of the experimental setup for the force measurement*

D. Apply the Flip Coil Technique

Construct an integrator circuit (Fig. 5) for use with the flip coil. Remember that R_1 in Fig. 5 is internal to the oscilloscope, so you only need to connect one resistor and one capacitor. Since you want the circuit time constant $t = RC$ to be much larger than the integration time t_1 you will need to use a relatively large capacitor. You would also want to keep RC not too large, since your signal depends inversely on this quantity. Try several values for R and C to find optimum working values.

Place the flip coil between the poles of the permanent magnet, and then quickly pull the flip coil away from the field, while monitoring the signal on the oscilloscope. Make sure that the plane of the coil is normal to the direction of the field, to avoid introducing a cos θ factor in your equations (θ is the misalignment angle). Record the peak voltage (at time t_1). This will be proportional to the field, as we can see from:

$$V_o(t) \approx \frac{A}{RC}\left(B(t) - B(0)\right)$$

which we get from Eq. 11 assuming that the field is approximately uniform over the area of the magnet poles and zero elsewhere. Since the area of the magnet pole face is smaller than that of the coil, A in the equation above represents the pole face area.

Repeat the experiment several times, so you will have a large number of V_0's. The distribution in these comes from a distribution in flip times. Use the average V_0 to estimate B and its error. Compare your best estimate to the value you got for the same magnet using the current balance technique.